Matroid covering games: the core and the nucleolus

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February 26, 2026

Abstract

The matroid bases covering problem, finding the minimum number of independent sets covering groundset in a matroid, is a classical topic in combinatorial optimization with wide applications in network design, resource allocation, and system reliability. This problem is dual to the well-studied matroid base packing problem, and together they form a foundational pair of covering and packing problems in matroid theory, with algorithmic roots tracing back to Edmonds' seminal work on matroid partition.

By examining the matroid covering covering problem from a cooperative game theoretic perspective, we introduce the matroid covering game to evaluate the role of each element in generating the minimum base covering. Briefly, the matroid covering game is a cooperative game defined on a matroid, where the players are the elements and the value of a coalition is the minimum number of independent sets covering the coalition. This formulation provides a natural way to evaluate the contribution of each individual element to the overall "covering capacity" of the system, offering a principled framework for fair allocation problems in matroid-based settings. Notably, matroid covering game generalizes the arboricity game, which corresponds to the case of graphic matroids.

In this paper, we study the core and the nucleolus of matroid covering game. The core is the solution that no coalition benefits by breaking away from the grand coalition and the nucleolus is the unique solution that lexicographically maximizes the vector of non-decreasingly ordered excess. Accordingly, we provide a complete characterization of the core for matroid covering games and develop an efficient algorithm for computing the nucleolus when the core is nonempty.

To establish these results, we provide a graphic characterization of the strong exchange properties of matroid bases. Using this characterization, we partition the ground set according to the strongly connected components of the underlying digraph. This leads to a complete characterization of the core. Moreover, agents from the same partition always have the same value in a core allocation. Consequently, when the core is not empty, this partition significantly reduces the number of variables and constraints required in the linear programs of Maschlers scheme and allows us to compute the nucleolus in polynomial time.

Keywords: Matroid · Matroid covering · Cooperative game · Core · Nucleolus